Government of **Western Australia**
School Curriculum and Standards Authority

Western Australian Certificate of Education Examination, 2015

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

Student Number: In figures ☐☐ ☐☐☐ ☐☐☐

In words ____________________

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time for this section:	one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	13	13	100	100	$66\frac{2}{3}$
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **66⅔% (100 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(5 marks)**

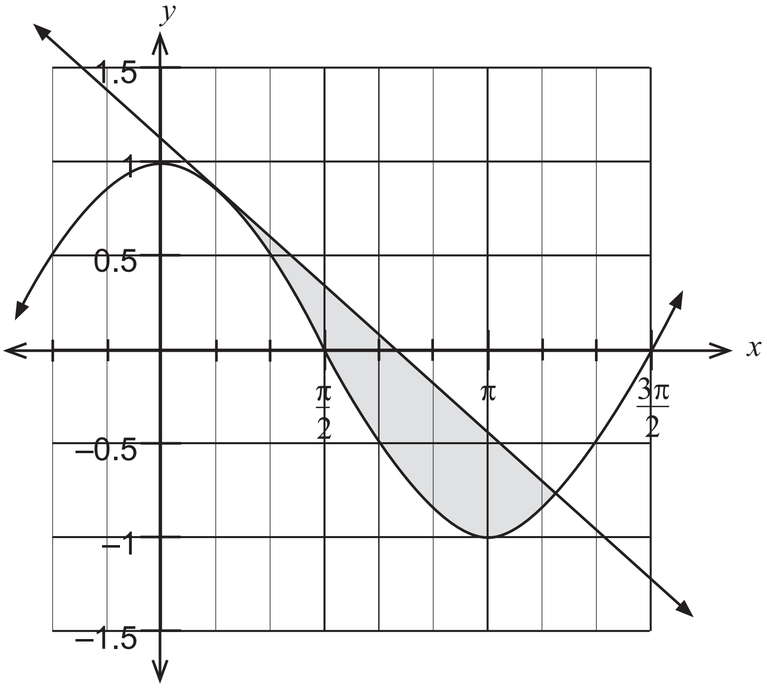
Consider the matrix $P = \begin{pmatrix} k+5 & 1 \\ 2 & k \end{pmatrix}$ where k is a real constant.

(a) If $P\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & -4 \end{pmatrix}$, solve for k. (2 marks)

(b) Determine whether P^{-1} exists for all values of k. (3 marks)

Question 10 **(8 marks)**

The tangent to the graph of $y = \cos x$ is drawn at $x = \frac{\pi}{6}$. This tangent intersects the graph of $y = \cos x$ again at $x = k$.



(a) Determine the equation of the tangent at $x = \frac{\pi}{6}$. (3 marks)

(b) Determine the value of k, correct to three decimal places. (2 marks)

(c) Write an expression for the area enclosed between the tangent at $x = \frac{\pi}{6}$ and the curve $y = \cos x$. (2 marks)

(d) Hence evaluate the area correct to two decimal places. (1 mark)

Question 11 **(6 marks)**

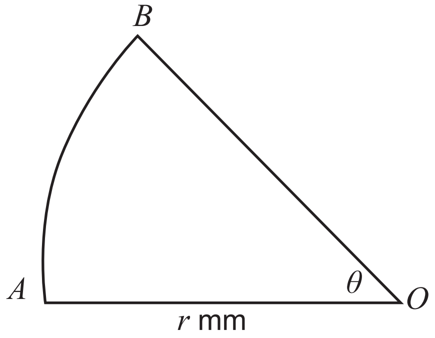
The line $\underset{\sim}{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the plane $\mathbf{r} \bullet (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 1$ intersect at point P.

(a) Determine the coordinates of P. (3 marks)

(b) Determine the size of the angle between the line and the plane. (3 marks)

Question 12 **(5 marks)**

The diagram below shows a sector OAB of a circle of radius r mm and centre O where $\angle AOB = \theta$.

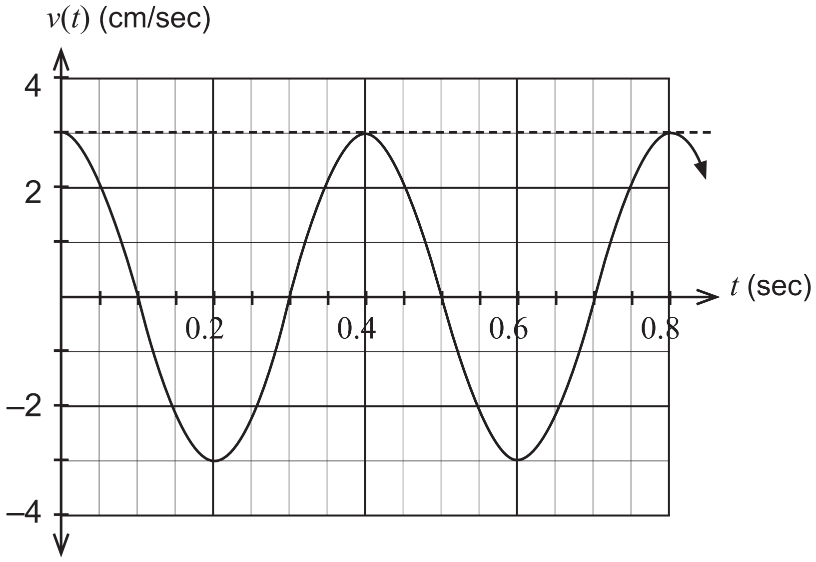


The value of r is increasing at the rate of 2 mm per second and the value of θ is increasing at the rate of 0.1 radians per second.

Find the rate of increase of the area of the sector when $r = 4$ mm and $\theta = \frac{\pi}{6}$, correct to two decimal places.

Question 13 **(10 marks)**

A particle performs simple harmonic motion with velocity $v(t)$, measured in cm per second, defined by the graph below.



(a) Given that the velocity $v(t) = 3\cos(bt)$, determine the value for b. (2 marks)

The displacement of the particle is given by $x(t)$ centimetres, with $x(0) = 1$.

(b) Determine $x(t)$ in the form $x(t) = A\sin(bt) + C$. (2 marks)

(c) Calculate, correct to two decimal places, how far the particle will travel in the first second. (3 marks)

(d) Show that $\frac{d^2x}{dt^2} = -k\,(x-1)$ for some appropriate value of k. (3 marks)

Question 14 **(8 marks)**

Transformation matrices A, B and S are defined by:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

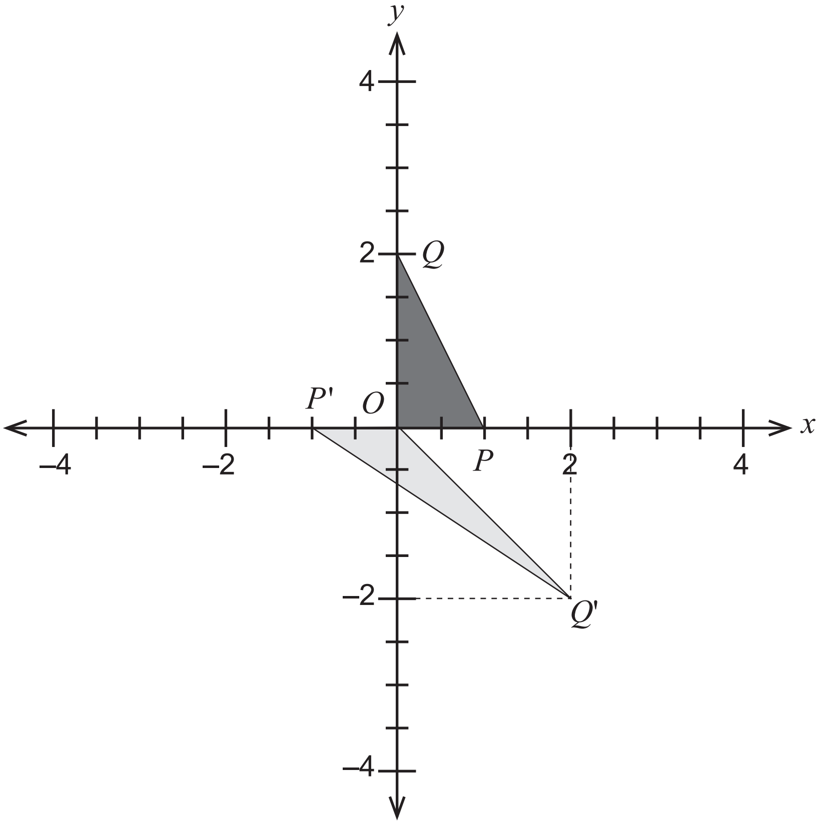
$S =$ a horizontal shear parallel to the x axis with factor equal to -1.

(a) Write S as a 2×2 matrix. (1 mark)

(b) Describe the transformation that is represented by the matrix AB. (3 marks)

(c) Determine the matrix R such that $R = B^2$. (1 mark)

Triangle OPQ is transformed by a matrix T to obtain $\Delta OP'Q'$ as shown in the diagram below. Point O is the origin.

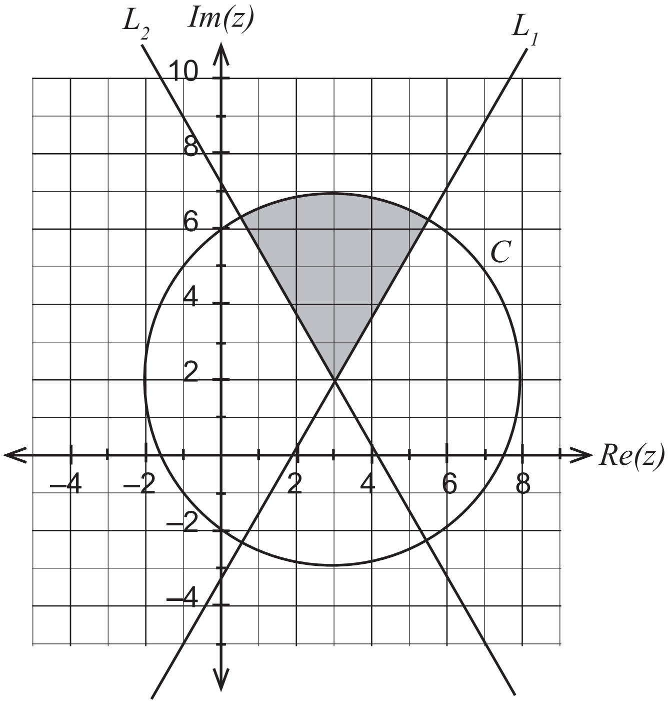


(d) Determine an expression for matrix T, in terms of two of the matrices A, B, R and S, so that ΔOPQ is transformed into $\Delta OP'Q'$. Provide appropriate reasoning for your answer.

(3 marks)

Question 15 **(7 marks)**

Two lines and a circle are shown below in the complex plane. Lines L_1 and L_2 intersect at $z = 3 + 2i$, while L_1 has a gradient of $\sqrt{3}$ and L_2 has a gradient of $-\sqrt{3}$.



(a) Write the equation of the circle C in the complex plane. (2 marks)

The equation of line L_1 can be written in the form $Arg\,(z - a - bi) = k$.

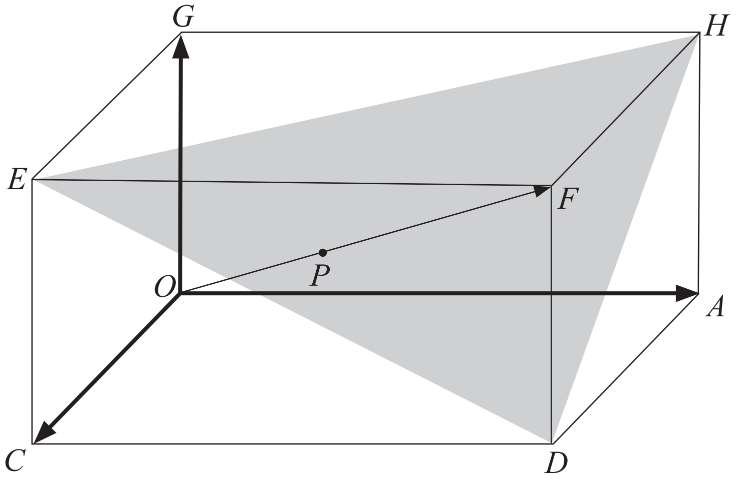
(b) State the values for a, b and k. (2 marks)

(c) Express the region shaded as the intersection of inequalities in the Argand plane. (3 marks)

Question 16 **(10 marks)**

The diagram below shows the positive coordinate axes $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OG}$, with point O being the origin. The following information is given.

- The points $O, A, D, C, E, F, H,$ and G form a rectangular prism.
- The points A, C and G have respective position vectors $\begin{pmatrix}4\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\2\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\3\end{pmatrix}$.
- The plane Π contains the points D, E and H.
- The vector $\overrightarrow{OF}$ intersects plane Π at point P.



(a) Determine the vector equation of the line that contains vector $\overrightarrow{OF}$. (1 mark)

(b) Explain why the equation for plane Π can be written in the form $\vec{r} = \overrightarrow{OD} + \mu\left(\overrightarrow{DE}\right) + \beta\left(\overrightarrow{EH}\right)$, where μ, β are any real numbers. (2 marks)

(c) Hence show that plane Π is given by $\boldsymbol{r} = \begin{pmatrix} 4 - 4\mu + 4\beta \\ 2 - 2\beta \\ 3\mu \end{pmatrix}$. (2 marks)

(d) Determine the position vector of point P. (2 marks)

(e) Show that the normal vector $\boldsymbol{n}$ for plane Π is parallel to $\begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix}$. (3 marks)

Question 17 **(7 marks)**

In the suburb of Peaceville, three supermarket chains compete for business. These supermarkets are K, S and W.

From week to week, there is a probability that shoppers will shop at the same supermarket or that they will shop at a different supermarket.

The matrix T indicates these probabilities. This matrix shows that if a customer shops at W one week, there is a 0.3 probability that they will shop at S the next week.

$$\begin{array}{cc} & \text{Current week} \quad \text{Next week} \\ & \begin{array}{ccc} \text{K} & \text{S} & \text{W} \end{array} \\ T = & \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.3 \\ 0.3 & 0.1 & 0.5 \end{pmatrix} \begin{array}{c} \text{K} \\ \text{S} \\ \text{W} \end{array} \end{array}$$

Suppose that initially, of all the customers in the suburb of Peaceville, 45% shop at K, 10% at S and 45% at W.

Let $P_0 = \begin{pmatrix} 0.45 \\ 0.1 \\ 0.45 \end{pmatrix} =$ the initial proportions matrix.

(a) Using matrices P_0 and T, write the matrix expression that will determine the proportions of customers who will be shopping at K, S and W two weeks later. (2 marks)

(b) Calculate the matrix expression from part (a). (1 mark)

(c) Given that a customer initially shopped at W, what is the probability that this customer shops at S two weeks later? (2 marks)

After many weeks, it is found that the supermarket system in Peaceville reaches equilibrium, where there is no change in the proportions matrix from week to week.

(d) Determine the equilibrium percentages of customers shopping at K, S and W. (2 marks)

Question 18 **(7 marks)**

Archie and Brianna are two radio-controlled drones carrying cameras. Archie leaves the point with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ kilometres at 2 pm, and travels with a constant velocity of $10\mathbf{i} - 10\mathbf{j} - 5\mathbf{k}$ kilometres per hour.

(a) Find the speed at which Archie moves. (1 mark)

(b) Calculate the position vector of the location of Archie at 4 pm. (1 mark)

The second radio-controlled drone, Brianna, leaves the point with position vector $-20\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}$ kilometres at 4 pm. Brianna travels with a constant velocity of $30\mathbf{i} - 30\mathbf{j} - 15\mathbf{k}$ kilometres per hour.

(c) Calculate the position vector of the location of Brianna at any time t. (1 mark)

(d) Find an expression for the vector representing the distance between Archie and Brianna at any time t. (1 mark)

(e) Use calculus to determine the shortest distance between Archie and Brianna. (2 marks)

(f) Correct to the nearest minute, at what time are they closest? (1 mark)

Question 19 **(9 marks)**

A type of lichen, L_1, is flat and circular, and grows continuously, always maintaining its circular shape with a fixed centre.

At any time t months, the instantaneous rate of growth of its surface area, $A \text{ cm}^2$, is $0.2A$.

(a) Write the formula for A, given the surface area of L_1 is 2 cm^2 when $t = 0$. (1 mark)

(b) Find the surface area of L_1 after three months. (1 mark)

Another type of lichen L_2 grows in such a way that its rate of increase of surface area is given by

$$\frac{dB}{dt} = 0.4\,B.$$

(c) Write the formula for B, given that the surface area of L_2 is 1.5 cm^2 when $t = 1$. (2 marks)

(d) When, to the nearest month, will L_1 and L_2 have the same surface areas? (2 marks)

(e) When will the lichens touch for the first time, given that their centres are 10 cm apart? (3 marks)

Question 20 **(8 marks)**

A number of fractions are multiplied together and the answers are tabulated.

n	Product $P(n)$	Answer $A(n)$
3	$\left(1-\frac{2}{3}\right)$	$\frac{1}{3}=\frac{2}{6}$
4	$\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)$	$\frac{1}{6}=\frac{2}{12}$
5	$\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)$	$\frac{1}{10}=\frac{2}{20}$
6	$\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\left(1-\frac{2}{6}\right)$	$\frac{1}{15}=\frac{2}{30}$
7	$\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\left(1-\frac{2}{6}\right)\left(1-\frac{2}{7}\right)$	$\frac{1}{21}=\frac{2}{42}$

(a) Write an expression for $P(8)$ and the answer $A(8)$. (2 marks)

(b) Determine for $n = 101$:

(i) the number of fractions multiplied to form $P(101)$. (1 mark)

(ii) the exact value for $A(101)$. (1 mark)

(c) Write an appropriate conjecture for $A(n)$, where $n \geq 3$. (1 mark)

(d) Use the method of induction to prove your conjecture. (3 marks)

Question 21 **(10 marks)**

A rumour spreads among 200 people at a rate given by $\frac{dN}{dt}$, where $N(t)$ is the number of people who have already heard it after t hours. Initially 40 people have heard the rumour.
It is known that $N(t)$ satisfies $Ae^{0.5t} = \frac{N}{200 - N}$, where A is some constant.

(a) Determine the value of A. (2 marks)

(b) By implicitly differentiating the equation $Ae^{0.5t} = \frac{N}{200 - N}$ with respect to t, show that $\frac{dN}{dt} = \frac{N(200 - N)}{400}$. (5 marks)

(c) Hence determine the fastest rate at which the rumour will spread. (3 marks)

End of questions

Additional working space

Question number: ________________

Additional working space

Question number: ________________

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